AP Calculus BC

Mean Value Theorem

1) $f(x) = -x^2 + 3x + 10$

a) $\frac{f(6)-f(2)}{6-2} = \frac{-8-(12)}{4}$

$= -5$

b) $f'(x) = -2x + 3$

$-2x + 3 = -5$

$\boxed{x = 4}$

2) a) $f(x) = x^2 - 5x + 2$ on $[-4, -2]$

+ $f(x)$ is cont on $[-4, -2]$
+ $f(x)$ is diff on $(-4, -2)$

MVT applies

$\frac{f(-2)-f(-4)}{-2-(-4)} = \frac{16-(38)}{2} = 11$

$f'(x) = 2x - 5$

$2x - 5 = 11$

$\boxed{x = 8}$

2) b) $y = \sin 3x$ on $[0, \pi]$

+ y is cont. on $[0, \pi]$
+ y is diff on $(0, \pi)$

MVT applies

$\frac{y(\pi)-y(0)}{\pi} = 0$

$y' = 3\cos(3x) = 0$

$\cos(3x) = 0$

$3x = \pi, 2\pi$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}}$

2) c) $y = (-5x + 15)^{1/2}$ on $[1, 3]$

+ y is cont. on $[1, 3]$
+ y is diff on $(1, 3)$

MVT applies

$\frac{y(3)-y(1)}{2} = \frac{0 - 10^{1/2}}{2} = \frac{\sqrt{10}}{2}$

$y' = \frac{1}{2}(-5x + 15)^{-1/2} \cdot (-5)$

$= \frac{-5}{2\sqrt{-5x+15}} = \frac{\sqrt{10}}{2}$

$\frac{2\sqrt{-5x+15}}{-5} = \frac{2}{\sqrt{10}}$

$\sqrt{-5x+15} = -\frac{5}{\sqrt{10}}$

$-5x + 15 = \frac{5}{2}$

$-5x = -\frac{25}{2}$

$\boxed{x = \frac{5}{2}}$

3) 2 times

in $[2, 5]$

$\frac{f(5)-f(2)}{5-2} = 4 \rightarrow f'(c) = 4$

And

in $[5, 11]$

$\frac{f(7)-f(5)}{2} = 6 \rightarrow f'(c) = 6 > 4$

$\frac{f(11)-f(7)}{11-7} = \frac{13}{4} < 4$

4) NONE

5) $h(x) = f(g(x)) + 2$

a) Since f & g are differentiable, h is differentiable.
h is continuous on $(2, 4)$

$h(2) = f(g(2)) + 2$
$= f(1) + 2$
$= 5 < 6$

$h(4) = f(g(4)) + 2$
$= f(3) + 2$
$= 7 > 6$

By IVT, $h(r) = 6$ in $(2, 4)$

b) $\frac{h(4) - h(2)}{4-2} = \frac{7-5}{2} = 1$

$h'(c) = 1$ by MVT.

6) $\frac{x(2) - x(0)}{2-0} = \frac{-1-1}{2} = -1$

$x'(t) = 3t^2 - 6t + 1 = -1$

$3t^2 - 6t + 2 = 0$

$t = 0.4226, 1.577$